

Experimental Demonstration of a Displacement Measurement of an Optical Beam beyond the Quantum Noise Limit *

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We experimentally generate a spatially squeezed light beam and realize a small-displacement measurement beyond the quantum noise limit with this squeezed light. Moreover, we measure about -2.2 ± 0.2 dB spatial squeezing and reduce the minimum measurable displacement from 1.17 \AA to 0.99 \AA with the signal-to-noise ratio normalized to 1.

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Optical displacement measurement is one of the important techniques in optical measurement. It can be used in many fields such as atomic force microscopy,^[1,2] high-resolution imaging,^[3] particle sensing and biological specimen measurement.^[4,5] In general, the precision of displacement measurement is limited by the shot noise limit (SNL). However, measurement beyond the SNL can be carried out by spatial squeezing and entanglement techniques.^[6–10] Treps *et al.* made a quantum laser pointer with -3.3 dB spatial squeezing, showing a factor of 1.5 precision improvement compared with the quantum noise limit. In addition, optimizing detection setups, such as by using homodyne detection instead of traditional split detection, can enhance the detection efficiency.^[11–13] Recently, a scheme of small-displacement measurements based on high-order Hermite–Gauss modes has been presented, proving that using high-order Hermite–Gauss modes can also enhance measurement precision.^[14] Meanwhile, parallel techniques on weak-value measurements have greatly improved the precision of beam deflection measurements.^[15,16]

In this Letter, we experimentally generate a spatially squeezed light beam by coupling the squeezed TEM₁₀ beam and an intense coherent TEM₀₀ beam on a 98/2 beam splitter. Using the TEM₁₀ homodyne detection scheme, we have measured -2.2 ± 0.2 dB spatial squeezing and have obtained the minimum measurable displacement of 0.99 \AA .

The electric field operator of a light beam with frequency ω can be written as its positive frequency part $E^+(x)$ by using slowly varying amplitude approximation in terms of photon annihilation operators^[11]

$$E^+(x) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0cT}} \sum_{n=0}^{\infty} \hat{a}_n u_n(x), \quad (1)$$

where $u_n(x)$ denotes the transverse amplitude distribution of the n th order Hermite–Gauss mode in one dimension, ω is the light frequency, c is the light speed in a vacuum, $2\pi\hbar$ is Planck's constant, ε_0 is the electric permittivity, T is the detection time and $\hat{a}_n = \langle \hat{a}_n \rangle + \delta\hat{a}_n$ is the annihilation operator of the n th mode.

For a TEM₀₀ beam, the mean photon numbers of the other high-order modes are all zero, while their quantum fluctuations are nonzero.^[10,11] After displacement of d for a TEM₀₀ beam, the electric field operator is given by

$$E^+(x-d) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0cT}} \left(\sqrt{N} u_0(x-d) + \sum_{n=0}^{\infty} \delta\hat{a}_n u_n(x-d) \right), \quad (2)$$

where N is the mean photon number of the signal field.

By using Taylor series expansion

$$u_0(x+d) = u_0(x) + u'_0(x)d + \dots \quad (3)$$

and the relation $u'_0(x) = -u_1(x)/w_0$, where w_0 is the beam waist of the signal mode, the signal field operator can be rewritten as

$$E^+(x-d) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0cT}} \left(\sqrt{N} \left(u_0(x) + \frac{d}{w_0} u_1(x) \right) + \sum_{n=0}^{\infty} \delta\hat{a}_n u_n(x-d) \right), \quad (4)$$

where the first order of d is reserved, omitting the other high orders terms.

We can see from Eq. (4) that the displacement of the TEM₀₀ mode mostly excites the TEM₁₀ mode. With TEM₁₀ homodyne detection, the displacement

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signal can be efficiently extracted. The TEM_{10} local oscillator operator is given by

$$E_{\text{LO}}^+(x, t) = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 cT}} \left(\sqrt{N_{\text{LO}}} u_1(x) + \sum_{n=0}^{\infty} \delta \hat{a}_n^{\text{LO}} u_n(x) \right), \quad (5)$$

where N_{LO} denotes the mean photon number of the local field, $\delta \hat{a}_n^{\text{LO}}$ is the fluctuation of the annihilation operator of the local field.

By using Eqs. (4) and (5) and omitting all the other small high orders terms, the photon number of BHD output is given by

$$\hat{n}_{-}^{\text{BHD}} = \sqrt{N_{\text{LO}}} \left(\frac{2\sqrt{N}}{w_0} d + \delta \hat{X}_1^+ \right), \quad (6)$$

where $\delta \hat{X}_1^+$ is the quadrature amplitude fluctuation denoted by $\delta \hat{X}_1^+ = \delta \hat{a}_1 + \delta \hat{a}_1^+$, which corresponds to the noise term with $n=1$ in Eq. (4), and $\delta \hat{a}_1$ and $\delta \hat{a}_1^+$ are the creation and annihilation operators of the TEM_{10} mode in the signal field, respectively.

The variance of BHD photon number is given by

$$V_{\hat{n}_{-}^{\text{BHD}}} = N_{\text{LO}} \left(\frac{4N}{w_0^2} d^2 + \delta^2 \hat{X}_1^+ \right), \quad (7)$$

where the first term is the displacement signal, and the second term is quantum noise.

In Eq. (7), for ordinary coherent light, $\delta^2 \hat{X}_1^+ = 1$ corresponds to the shot noise limit. If we use the TEM_{10} mode squeezed noise to fill the vacuum channel of the signal field, the measured noise will be squeezed, i.e., $\delta^2 \hat{X}_1^+ < 1$, and the signal light is called spatially squeezed light at $d = 0$. This corresponds to a displacement measurement beyond the shot noise limit.

In the experiment, imperfect factors need to be considered, such as the mode matching efficiency $\eta_1 = \xi_1^2$ between the TEM_{00} mode and the squeezed TEM_{10} mode on the 98/2 splitter, homodyne efficiency $\eta_2 = \xi_2^2$ on the 50/50 splitter and the quantum efficiency η_3 of homodyne detectors, as shown in Fig. 1. The symbols ξ_1 and ξ_2 denote the corresponding interference visibilities. Considering these efficiencies,^[13] the signal-to-noise ratio of the displacement measurement is given by

$$\text{SNR} = \eta_t \frac{4N}{w_0^2} d^2 / \delta^2 \hat{X}_1^+, \quad (8)$$

where $\eta_t = \eta_1 \eta_2 \eta_3$ is the total efficiency.

The minimum measurable displacement can be defined as the displacement with $\text{SNR}=1$

$$d_{\text{min}} = \frac{w_0}{2\sqrt{\eta_t} \sqrt{N}} \sqrt{\delta^2 \hat{X}_1^+}. \quad (9)$$

The mean photon number in the experiment can be expressed as

$$N = \frac{1}{\text{RBW}} \frac{P_s \lambda}{2\pi \hbar c}, \quad (10)$$

where P_s is the light power of the signal beam, λ is the light wavelength, and RBW is the resolution bandwidth related to the electronic spectrum analyzer.

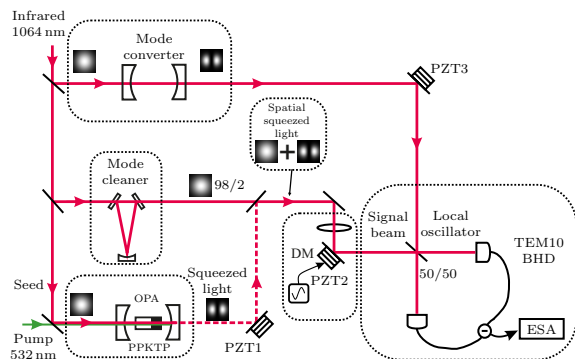


Fig. 1. (Color online) Experimental setup: OPA: degenerate optical parametric amplifier, DM: displacement modulation, PZT: piezoelectric ceramic transducer, ESA: electronic spectrum analyzer, BHD: balanced homodyne detection.

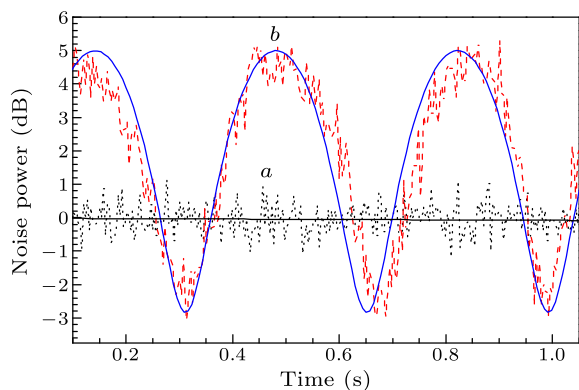


Fig. 2. (Color online) The amplitude squeezing of the TEM_{10} mode versus the sweeping time.

The experimental setup is shown in Fig. 1. We use a continuous-wave (CW) intracavity frequency doubled Nd:YAG laser as the light source, which provides 80 mW of 1064 nm infrared light and 1 W of 532 nm green light. The infrared seed wave and green pump wave are injected into the OPA to generate the amplitude squeezed light of the TEM_{10} mode. A mode cleaner is used to obtain a TEM_{00} mode with low thermal noise and optimized transverse profile. A TEM_{00} mode intense coherent beam and the TEM_{10} mode amplitude squeezed beam are coupled on a 98/2 beam splitter (BS) to produce a spatially squeezed light beam. The displacement modulation is realized by a mirror mounted on a PZT. The local beam is a TEM_{10} mode in balanced homodyne detection (BHD). The final signal from BHD is analyzed by an electronic spectrum analyzer (ESA). The TEM_{10} mode local oscillator beam in homodyning comes from a mode con-

verter, which transforms the TEM_{00} mode into the TEM_{10} mode.

The first step of the experiment is to generate TEM_{10} quadrature amplitude squeezing with OPA under the parametric de-amplification condition.^[17] Blocking the intense light from the mode cleaner and turning off displacement modulation, we can obtain a TEM_{10} quadrature amplitude squeezing. The results are plotted in Fig. 2, with the local phase swept in time. Lines (a) and (b) in Fig. 2 show the shot noise limit (SNL) and the noise power, respectively. The dashed and the dotted lines are the experimental plots while the solid lines are their theory fittings. About -2.7 ± 0.2 dB amplitude squeezing is measured at analyzing frequency 3 MHz. The pump threshold is nearly 400 mW for the TEM_{10} mode. Compared with TEM_{00} signal mode squeezing generation, it is very high due to the fact that the pump mode is not the optimized mode in a profile for generating TEM_{10} mode squeezing while it is simple enough.^[18] The resolution bandwidth and video bandwidth of ESA are 30 kHz and 300 Hz, respectively. The output power of OPA is about $5 \mu\text{W}$ and the local power is 1.4 mW. The interference visibility for the TEM_{10} mode is $\xi_2 = 0.95$. The quantum efficiency of the detection system is $\eta_3 = 0.91$. Considering the efficiency $\eta = 0.91 \times 0.95^2 = 0.82$, the actual noise variance from OPA is $V_{\text{sq,th}} = (V_{\text{sq,det}} + \eta - 1)/\eta$,^[19] and therefore, the inferred squeezing is -3.6 dB.

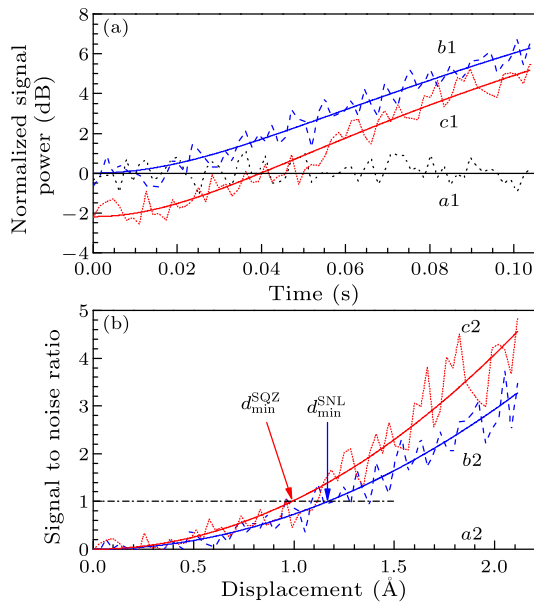


Fig. 3. (Color online) (a) The variance of normalized signal power versus sweep time. (b) Signal-to-noise Ratio versus displacement.

Opening the intense TEM_{00} beam from the mode cleaner, we obtain a spatially squeezed light. We also realized the beam displacement measurement beyond SNL with the spatially squeezed light. The measurement results are plotted in Fig. 3. The signal power versus sweeping time is plotted in Fig. 3(a). The hor-

izontal axis corresponds to beam displacement. Trace (a1) is the shot noise limit without squeezing or displacement modulation. Trace (b1) is the coherent light displacement measurement without squeezing while with displacement modulation. Trace (c1) is the same measurement by using spatially squeezed light. The solid lines correspond to theory fittings.

The measured power is the sum of the displacement signal power and noise power. At the beginning in Fig. 3(a), it is only the noise power for coherent and spatially squeezed light since the displacement is zero. It means that we have obtained -2.2 dB spatially squeezed light. It is smaller than TEM_{10} amplitude squeezing of -2.7 dB in Fig. 2 due to the noise of the TEM_{00} signal mode introduced on the 98/2 beam splitter. With the sweep time increasing, the measured power increases both with coherent and spatially squeezed light. The corresponding signal-to-noise ratio (SNR) versus the displacement is shown in Fig. 3(b). Trace (a2) corresponds to SNR=0 without a modulation signal. Traces (b2) and (c2) correspond to coherent light and spatially squeezed light, respectively. The solid lines are the corresponding theory fittings. With the displacement increasing, the SNRs increase. The trace (c2) with spatially squeezed light increases rapidly compared with trace (b2).

The minimum measurable displacement is defined as the SNR=1. For coherent light measurement, the minimum measurable displacement is given by

$$d_{\text{min}}^{\text{SNL}} = \frac{w_0}{2\sqrt{\eta_t}\sqrt{N}} \approx 1.17, \quad (11)$$

where $w_0 = 21 \mu\text{m}$ is the waist of the TEM_{00} mode Gaussian beam at the position of the displacement modulation, $\eta_t = \eta_1\eta_2\eta_3 = 0.91 \times 0.95^2 \times 0.95^2 = 0.74$ is the total efficiency, and N is the detected photon number calculated with Eq. (10), where $\lambda = 1064 \text{ nm}$, $P_s = 62 \mu\text{W}$ and $\text{RBW} = 30 \text{ kHz}$.

As shown in Fig. 3(b), for the spatially squeezed light, the minimum measurable displacement in the experiment is $d_{\text{min}}^{\text{SQZ}} = 0.99 \text{ \AA}$, surpassing the shot noise limit and signifying a better measurement.

In conclusion, we experimentally generate a spatially squeezed light beam. Using this nonclassical beam, we demonstrate the displacement measurement beyond the quantum noise limit and the minimum measurable displacement is improved from 1.17 \AA to 0.99 \AA . This nonclassical light has been used in spatial variable measurement, biological measurement, and has great potential applications in quantum information transferring^[20] and even gravitational wave detection.^[21]

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